DOCUMENT RESUME

ED 293 727 SE 049 114

AUTHOR Bradley, Carol A.
TITLE The Relationship between Mathematics Language Facility and Mathematics Achievement among Junior High School Students.
PUB DATE 88
NOTE 33p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 5-9, 1988). Light type and charts may not reproduce well.
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Classroom Communication; Junior High Schools; *Junior High School Students; *Language Acquisition; Mathematical Linguistics; *Mathematical Vocabulary; *Mathematics Achievement; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Secondary Education; *Secondary School Mathematics
IDENTIFIERS Mathematics Education Research

ABSTRACT

This paper argues that two serious flaws are present in current mathematics learning and instruction: (1) the lack of meaning in students' mathematical knowledge; and (2) students' inability to communicate using appropriate mathematical language and ideas. The study explored the relationship between mathematics language facility and mathematics achievement. Written tests in mathematics language and mathematics achievement were administered to 177 junior high school students. Results of ANOVAs indicated three groups in the sample: average minority, average majority, and above-average students. Multiple regression analyses showed that, for both average groups, the "interaction" of language facility and procedural knowledge significantly predicted conceptual achievement. For above-average students, language facility and procedural knowledge each significantly predicted conceptual achievement. The relationship was further explored by comparing conceptual achievement means for combinations of high and low performance in mathematics language and procedures. Results clearly illustrated the interaction between language and procedure for both average groups. For above-average students, the additive effects of the two predictors were evident. A 50-item reference list is included. (Author/PK)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED293727

The Relationship Between Mathematics Language Facility and Mathematics Achievement Among Junior High School Students

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Carol A. Bradley

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

by

Carol A. Bradley

University of California, Los Angeles

Hoffman Educational Systems, Duarte, CA

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Paper presented at AERA Annual Meeting, April 3, 1988)

SE 049 114

BEST COPY AVAILABLE

Acknowledgements

I wish to thank Noreen Webb for her continual support and assistance during the research project and in the preparation of this paper.

The Relationship Between Mathematics Language Facility and Mathematics Achievement Among Junior High School Students

Carol A. Bradley
University of California, Los Angeles
Hoffman Educational Systems, Duarte, CA

Two serious flaws are present in current mathematics learning and instruction: the lack of meaning in students' mathematical knowledge and students' inability to communicate using appropriate mathematical language and ideas. This study explored the relationship between mathematics language facility and mathematics achievement. Written tests in mathematics language and mathematics achievement were administered to 177 junior high school students. Results of ANOVAs indicated three groups in the sample: average minority, average majority and above-average students. Multiple regression analyses showed that, for both average groups, the interaction of language facility and procedural knowledge significantly predicted conceptual achievement. For above-average students, language facility and procedural knowledge each significantly predicted conceptual achievement. The relationship was further explored by comparing conceptual achievement means for combinations of high and low performance in mathematics language and procedure. Results clearly illustrated the interaction between language and procedure for both average groups. For above-average students, the additive effects of the two predictors were evident.

The Relationship Between Mathematics Language Facility and Mathematics Achievement Among 7th and 8th Graders

Two serious flaws are present in current mathematics learning and instruction. The first is the lack of meaning in students' mathematical knowledge. Many students learn mathematics primarily on a procedural level and do not understand the meaning of mathematics concepts, nor how to apply them (Carpenter et al., 1980; Hiebert and Wearne, 1985; NAEP, 1983). This gap between procedural knowledge and conceptual understanding is apparent across grade levels. For example, at the elementary school level, a discrepancy between performance of computation tasks (i.e. adding, subtracting, multiplying and dividing numbers) and understanding the conceptualization of the computation (i.e., writing number sentences and knowing what the algorithms mean) has been reported (Carpenter et al., 1982, 1983; Davis and McKnight, 1980; Hendrickson, 1979; Resnick and Ford, 1981; Sleznick, 1982). In secondary and higher education, incomplete conceptual understanding in algebra has been reported for high school students (Matz, 1980) and university students (Resnick and Clement, 1980) who have procedural skill but lack understanding of the meaning of equations and relationships.

The second flaw in mathematics education is students' inability to communicate with appropriate mathematical language

and ideas. Their communication skills in mathematics are generally inadequate, incomplete and sometimes inappropriate (Skypek, 1982). The paucity of meaningful, literate conversation and expression of ideas in mathematics classrooms has a long history in American education (Goodlad, 1984; Hoetker and Ahlbrand, 1969). Moreover, just like the discrepancy between procedural and conceptual knowledge described earlier, the lack of meaningful verbal interaction in mathematics classrooms is evident across grade levels (Bishop, 1983; Clement, 1982; Davis, 1984; Dudley, 1984; Fey, 1970; Peterson et al., 1982; Peterson and Swing, 1982; Wollman, 1983). This study investigated the possible connection between the lack of conceptual learning in mathematics and the lack of mathematics language facility.

Theories about the relationship between language and concept acquisition (Inhelder and Piaget, 1964; Siegel, 1971, 1982; Tharp et al., 1984) generally agree that language exerts a strong influence on cognition. These theories have grown out of work with young children. There has been little research, however, on the relationship between language and concept development among junior high school students learning mathematics. The purpose of this study was to explore the relationship between mathematics language facility and mathematics achievement among 7th and 8th graders. While this paper does not attempt to determine causality in the link between language facility and

achievement in mathematics, it does shed light on the relationship between them.

Existing research on the relationship between mathematics language and mathematics achievement focuses on two areas. First, studies investigating the role of oral language use have shown that verbalizing strategies and procedures is a common practice during problem-solving (Lesh, Landau and Hamilton, 1983), leads to fewer errors and increased performance speed (Gagne and Smith, 1962) and promotes active cognitive processing (Lewis and Anderson, 1985). Webb (1980a, 1980b, 1980c; 1982a, 1982b; 1983; 1984) found that giving and receiving explanations while solving mathematical problems in small groups was related to higher achievement for junior high and high school students, even when controlling for ability.

The second area of research has explored the importance of understanding the language used in mathematics problems and texts. Several studies reported that understanding the language in problems and books related to higher student achievement (Caldwell and Goldin, 1979; Hiebert, 1982; Knight and Hargis, 1977; Linville, 1976; Watkins, 1979). Other studies have addressed specific factors related to understanding math language. In a study of 6th graders' comprehension of common and mathematical words in their text, Earp and Tanner (1980) found that while students' decoding skills were adequate, comprehension

of math words was poor. However, when meaningful contextual clues were provided, students' comprehension of math terms improved. Earp and Tanner suggested that oral language activities may promote better understanding of mathematics vocabulary. Knight and Hargis (1977) hypothesized that specific language inadequacies, such as misunderstanding mathematics grammar and phrases, can lead to learning difficulties.

In summary, previous research reported that verbalizing mathematical ideas and procedures relates to improved problem solving performance, and understanding the context and structure of mathematics language affects comprehension of mathematical text. However, studies have not sufficiently addressed the relationship between mathematics language facility and conceptual understanding in mathematics.

The relationship between mathematical language and mathematical thought is important to investigate, both from a learning and an instructional point of view. Recent research on elaboration and learning (Mayer, 1980; Weinstein and Mayer, 1986) has shown that language activities, such as explaining and writing, encourage learners to relate existing knowledge to new information. Without knowledge integration, learning remains on a rote level (Walker and Meyer, 1980). Regarding instruction, Wittrock (1974; 1986) has shown that when explicit instructional activities are used to encourage students to generate their own

language and ideas, comprehension is increased. Traditionally, mathematics learning and instruction have not involved explicit language activities for students.

The purpose of this study is to explore factors involved in conceptual understanding in mathematics. It investigates relationships among students' mathematics language facility, their procedural knowledge in mathematics, and their understanding and application of mathematics concepts.

Method

Subjects

71 boys and 106 girls in grades 7 and 8 participated in the study (six intact classes of three math teachers). Four ethnic groups were represented in the sample: 84 Whites, 10 Blacks, 64 Hispanics, and 19 Asian-Americans. Students' ability levels ranged from "low-average" to "above-average".

Measures

Background Achievement. Reading, Language, Math Computation and Math Concepts scores on the Comprehensive Test of Basic Skills (CTBS) Form U, Level G, were collected from cumulative records for each student. Results of a short screening test verified that all students had knowledge of basic number facts, ordering numbers and using operations.

Written Tests. Two types of fraction tests were developed for this study: a math achievement test and a math language test.

The math achievement test had two parts: it measured procedural knowledge and conceptual knowledge of fractional numbers. The procedural items tested Basic Skills (recognition, comparison, equivalent forms), Operations (+, −, ×) and Estimation (+, −, ×). The conceptual items measured Concepts (meaning of fractional numbers, comparison without computation, meaning of equivalent fractions) and Application.

The math language test had three parts: Vocabulary (specific, operational/relational, names), Phrases and Problems (understanding math terms in context), and Expression (using math language in a written explanation of a math procedure). Internal consistency alpha coefficients for the procedural math measures are the following: Basic Skills, .74; Operations, .78; Estimation, .57. For the conceptual knowledge measures, reliability coefficients are: Concepts, .81; and Application, .70. Internal consistency coefficients for the language measures are: Vocabulary (used as a total score), .72; Phrases, .52; Problems, .49; and Expression, .57.

Procedure

Each class was given the mathematics achievement test and the mathematics language test on two consecutive days. All students completed each test in one math period (50-55 minutes).

Preliminary Analyses

Preliminary analyses were conducted to identify significant differences in gender, ethnicity and ability. Results of Analyses of Variance showed significant differences on most achievement variables for three groups: average minority students (n = 52), average majority students (n = 55), and above-average students (n = 70). These three groups were distinguished in all subsequent analyses.

Further analyses were conducted to determine which variables should be combined into composites. Because the background ability variables (Reading, Language, Computation and Math Concepts) were highly intercorrelated (correlations ranged from .27 to .70), the four measures were each standardized and then combined to form a background ability composite. Similarly, a Language Facility composite was created by combining Vocabulary, Phrases, Problems and Expression (correlations ranged from .10 to .54) and a procedural Knowledge composite was formed by combining Basic Skills, Operations and Estimation (correlations ranged from .22 to .56). The conceptual knowledge variables, Concepts and Application, were analyzed separately because the correlations between them were low for the two average groups (.15 and .16 for minority and majority groups, respectively). Although the correlation between the two conceptual variables was substantial for the above-average group (.71), they were kept separate in

this group to maintain consistent analyses across the three groups.

Results

The purpose of this study was to explore factors involved in conceptual understanding in mathematics. It investigated the relationships among mathematics language facility, procedural knowledge in mathematics, and understanding and application of mathematics concepts.

Relationships Among Mathematics Language, Procedural Knowledge, and Conceptual Knowledge. The correlations, means and standard deviations among composites are presented in Table 1. For the average minority group, the language facility composite correlated significantly with procedural knowledge but not with conceptual knowledge (Concepts and Applications). For the average majority students, mathematics language facility significantly correlated with both procedural and conceptual knowledge. And, for the above-average students, language facility was sigificantly related to both procedural and conceptual knowledge in mathematics. These correlations suggest that mathematics language and achievement are interrelated. The next section clarifies this relationship for each group of students.

The Best Predictors of Conceptual Understanding in Mathematics. To determine the best predictors of conceptual

Table 1
Means, Standard Deviations and Correlations for Composites

	M	SD	1	2	3	4
Average Minority						
1. Background Ability	11.16	2.33	----			
2. Procedural Math	4.30	1.54	.43*	----		
3. Concepts	11.29	5.21	.37*	.49**	----	
4. Application	2.02	1.87	.26	.37*	.15	----
5. Language Facility	9.97	2.19	.43*	.39*	.32	.23
Average Majority						
1. Background Ability	12.50	2.29	----			
2. Procedural Math	4.98	1.94	.66**	----		
3. Concepts	12.67	5.66	.40*	.48**	----	
4. Application	2.96	2.02	.20	.41*	.16	----
5. Language Facility	11.30	2.55	.51**	.57**	.43**	.34*
Above-Average						
1. Background Ability	16.30	3.05	----			
2. Procedural Math	8.02	2.05	.60**	----		
3. Concepts	21.79	8.43	.60**	.70**	----	
4. Application	6.24	2.83	.43**	.63**	.70**	----
5. Language Facility	14.08	2.59	.74**	.56**	.66**	.56**

* p < .01.
** p < .001.

understanding, multiple regression analyses were run predicting the two conceptual achievement variables, Concepts and Application, from the background ability composite, the procedural knowledge composite and the language facility composite. For these analyses, not only were the procedural knowledge and language facility composites used as separate predictors, but the interaction between them was included in each regression analysis. This interaction was of interest because previous research has shown that language and procedural math knowledge are each related to conceptual understanding, but the effect of the combination of both has not been investigated.

The results of the regression analyses are presented in Tables 3, 3, and 4. In both average groups, the interaction between procedural mathematics achievement and mathematics language facility significantly predicted knowledge of mathematics concepts (Concepts). In the average minority group, the interaction contributed 22% to the R-square predicting Concepts. In the average majority group, the interaction added 15% to the R-square predicting Concepts. These results show that conceptual understanding is not a simple additive function of procedural knowledge and language facility. Rather, the combination of both is a powerful predictor of conceptual achievement. The interaction of procedural math and math language facility did not significantly predict Application in

Table 2
Multiple Regression Analyses Predicting Conceptual Math Achievement (Average Minority)

Predictor	b	s.e.
Outcome = Concepts		
Background Ability	Ø.43	Ø.3Ø
Procedural Math Achievement	-6.66**	2.Ø4
Math Language Facility	-2.57**	Ø.77
Interaction of Procedure and Language	Ø.71***	Ø.18
Outcome = Application		
Background Ability	Ø.Ø7	Ø.14
Procedural Math Achievement	1.11	Ø.94
Math Language Facility	Ø.33	Ø.34
Interaction of Procedure and Language	-Ø.Ø7	Ø.Ø8

* p < .Ø5.
** p < .Ø1.
*** p < .ØØ1.

Table 3
Multiple Regression Analyses Predicting Conceptual Math Achievement (Average Majority)

Predictor	b	s.e.
Outcome = Concepts		
Background Ability	Ø.28	Ø.4Ø
Procedural Math Achievement	-3.88*	1.5Ø
Math Language Facility	-1.49*	Ø.68
Interaction of Procedure and Language	Ø.4Ø**	Ø.12
Outcome = Application		
Background Ability	-Ø.15	Ø.16
Procedural Math Achievement	-Ø.37	Ø.62
Math Language Facility	-Ø.19	Ø.28
Interaction of Procedure and Language	-Ø.Ø7	Ø.Ø5

* $p < .Ø5$.
** $p < .Ø1$.
*** $p < .ØØ1$.

either the average minority or average majority group.

The interaction was not a significant predictor of either Concepts or Application in the above-average group. Procedural knowledge and language facility were, however, significant main effects in predicting both Concepts and Application for these students. In the above-average group, then, an additive model was sufficient to explain conceptual understanding.

To illustrate the importance of the interaction between procedural knowledge and mathematics language facility for conceptual understanding, each group was split into four subgroups of every combination of high and low performance on procedural knowledge and language facility. The split between high and low was determined on the basis of the median of the group involved. The resulting subgroups were: low procedure/low language, low procedure/high language, high procedure/low language, high procedure/high language.

The mean score and standard deviation in Concepts and Application were computed for each subgroup and Analyses of Variance were run to determine which subgroup means were significantly different (Table 5 and Figure 1). Significant differences appeared in four out of six tests. Results of post hoc comparisons using Tukey's HSD test ($p<.05$) showed that the high procedure/high language subgroups in both groups of average students were significantly different from the other subgroups in

Table 4

Multiple Regression Analyses Predicting Conceptual Math Achievement (Above-Average)

Predictor [a]	b	s.e.
Outcome = Concepts		
Background Ability	Ø.19	Ø.38
Procedural Math Achievement	1.93***	Ø.46
Math Language Facility	1.12*	Ø.44
Outcome = Application		
Background Ability	-Ø.16	Ø.14
Procedural Math Achievement	Ø.7Ø***	Ø.18
Math Language Facility	Ø.44**	Ø.17

[a] For each outcome, the interaction was not entered into the equation due to tolerance limits being reached (.Ø1Ø).

* $p < .Ø5$.

** $p < .Ø1$.

*** $p < .ØØ1$.

Table 5
Mean Scores in Conceptual Knowledge by Procedure and Language Subgroup

	n	M	SD
Average Minority			
Concepts			
Low Procedure/Low Language	13	9.9	3.9
Low Procedure/High Language	10	9.7	5.1
High Procedure/Low Language	10	7.8	2.8
High Procedure/High Language	14	16.4	4.3
Application			
Low Procedure/Low Language	12	1.0	1.1
Low Procedure/High Language	10	2.0	1.6
High Procedure/Low Language	10	2.3	2.3
High Procedure/High Language	13	2.7	1.8
Average Majority			
Concepts			
Low Procedure/Low Language	15	10.0	3.7
Low Procedure/High Language	6	8.5	3.3
High Procedure/Low Language	10	11.5	5.3
High Procedure/High Language	17	17.4	4.8
Application			
Low Procedure/Low Language	15	2.1	1.8
Low Procedure/High Language	6	3.2	1.5
High Procedure/Low Language	9	3.2	2.3
High Procedure/High Language	17	3.5	2.2
Above-Average			
Concepts			
Low Procedure/Low Language	22	15.1	5.9
Low Procedure/High Language	9	22.2	10.6
High Procedure/Low Language	8	22.2	5.2
High Procedure/High Language	21	28.3	5.2
Application			
Low Procedure/Low Language	22	4.3	2.7
Low Procedure/High Language	8	7.4	2.9
High Procedure/Low Language	8	5.9	1.9
High Procedure/High Language	21	7.8	1.5

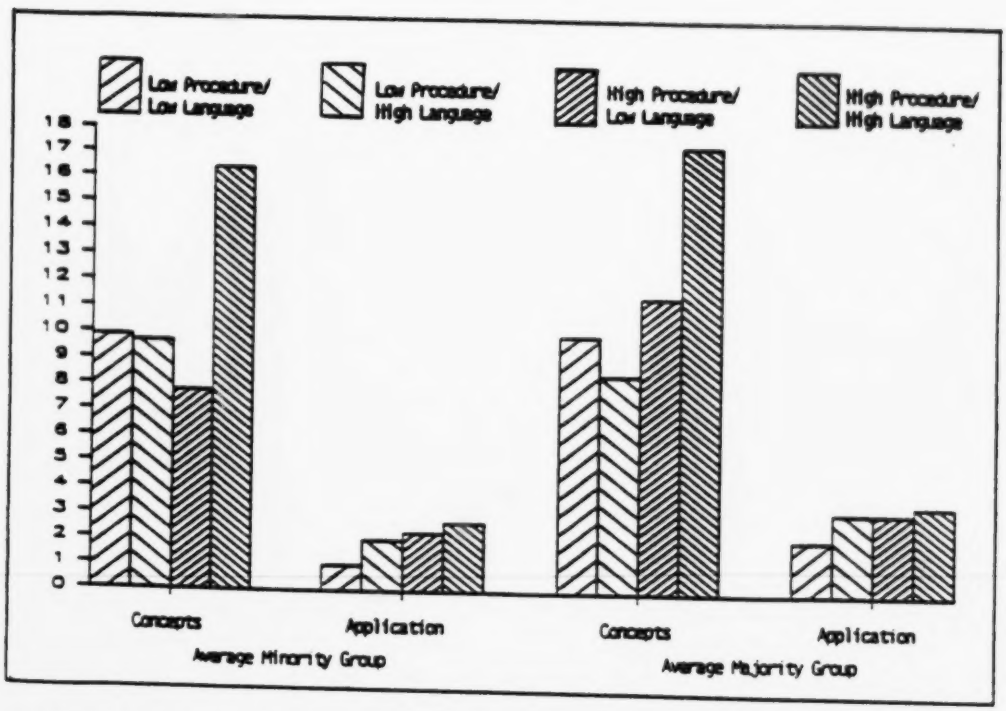

FIGURE 1. Mean Scores in Conceptual Knowledge by Procedure and Language Subgroup for Both Average Groups.

Concepts. There were no significant differences among subgroups for average students in Application. For above-average students, the low procedure/low language subgroup was significantly different from all subgroups in Concepts, except for high procedure/low language students. In Application, the low procedure/low language subgroup was significantly different from all other subgroups of above-average students.

The impact of the interaction between procedural knowledge and language facility on students' knowledge of Concepts for the average minority and average majority groups is best seen in Figure 1. Having either high procedural knowledge or high language facility did not add much to students' conceptual knowledge compared to students who were low on both. But students who scored high on both variables scored significantly higher in conceptual knowledge than the other three subgroups.

The lack of any significant effect of Procedural Math and Language Facility on Application in the two average groups is also illustrated in Figure 1. The means in all four subgroups are nearly equal. Application can not be predicted from either procedural knowledge or mathematics language facility separately or in combination.

For the above-average group (Table 5), the additive effects of Procedural Math and Language Facility-and the absence of an interaction-are apparent. Compared to being low in procedure and

language, having a high score on either Procedural Math or Language Facility adds about 7 points to students' scores on Concepts. It doesn't matter in which area a student is proficient, procedural knowledge or language facility, to add 7 points. Being proficient in both procedure and language adds about 13 points, approximately the same as the sum of the effects of Procedural Math and Language Facility separately. Knowing the separate effects of being high on Procedural Math or being high on Language Facility is sufficient to predict the mean of being high on both. The same pattern applies to Application.

In summary, these results show that the interaction of procedural knowledge and language facility significantly predicted conceptual understanding for the average minority and the average majority groups. For both groups, the <u>combination</u> of procedural knowledge and language facility was the best predictor of knowledge of Concepts. High performance on Concepts required <u>both</u> high performance on Procedural Math and high performance on Language Facility. Performing well on just one of the two variables was not sufficient to significantly increase performance on Concepts. For the above-average group, in contrast, each composite was a significant predictor of conceptual knowledge, but the interaction was not. Performing well on either Procedural Math or Language Facility predicted performance on both Concepts and Application for this group.

Discussion

This study showed that the interaction of procedural knowledge and language facility in mathematics significantly contributed to conceptual achievement, at least for students in average classes. Students who had either high procedural knowledge or high language facility, but not both, showed little increase in conceptual knowledge over the students who were low in both mathematics procedure and language. However, those students who were high on both procedural knowledge and language facility exhibited significantly more conceptual knowledge on written measures of performance.

For students in the above-average classes, procedural math knowledge and math language facility each were significant predictors of conceptual math achievement, but only in additive fashion, not in combination. Specifically, students who were low in both procedural math knowledge and math language facility were also low in conceptual understanding. Proficiency in either mathematics procedure or mathematics language contributed about the same to conceptual understanding. Those students who were high in either procedural knowledge or language facility showed about the same amount of increase in conceptual knowledge over the students in the low procedure/low language category.

These findings indicate that students needed both procedural knowledge and language facility in mathematics for an acceptable

level of conceptual achievement. As discussed below, these results have several practical and theoretical implications for learning and instruction in mathematics.

Implications of Findings for Classroom Practice. Typical mathematics instruction does not involve language development activities, but rather focuses on the development of procedural knowledge and skill. Rarely are students required to give reports, have group discussions, write and talk about their ideas, hold meaningful conversations about important concepts or illustrate their understanding of concepts in varied ways. In mathematics classes, these activities, if used at all, are often used for "extra credit". In contrast, instruction in other subjects generally includes language development as a critical learning activity. The role of language facility in mathematics learning is highlighted by the findings of this study which showed that math language skills, in conjunction with procedural skills, were related to conceptual understanding. While causal direction was not determined, the results of this investigation show that there is a relationship between students' language facility and their conceptual achievement.

The possible interpretation that language facility is important for higher mathematics achievement suggests that math language development needs to be investigated as a variable in mathematics concept acquisition. Perhaps providing math language

activities during instruction can help students bridge the gap between procedural knowledge and conceptual understanding. As this study showed, without language facility, students' procedural knowledge alone did not predict conceptual understanding. This interpretation suggests that mathematics teachers should incorporate mathematics language development into current teaching practices. For example, class time spent on drills, quizzes and homework-checking can occasionally be used for class discussions on topics such as: variations in problem solving techniques, different explanations of difficult concepts, derivations of mathematical words, and causes of common misunderstandings. Homework and "seatwork" can be varied to include written and oral descriptions of solutions and ideas or explanations of relationships between previously learned topics and new ones. Nahrgang and Petersen (1986) described a project in which high school mathematics students kept weekly journals to express their ideas and understanding of topics being learned. Teachers observed that journal writing surpassed written assessments (e.g. quizzes) in encouraging students to associate problem solving with mathematics concepts.

Implications for Learning Theory. The findings in this study raise the question of why the interaction of mathematics language facility and procedural knowledge in mathematics related to conceptual understanding for the average students but not for

the above-average students. It appeared that the above-average students could rely on either understanding of mathematics language or good procedural knowledge for conceptual achievement. Whether this flexibility is attributable to their specific strengths in mathematics language and procedure or to their higher general ability cannot be determined by this study. But, the distinction is important because it impacts on when and how the interaction should be mediated. That is, the interaction may be essential during early development of language proficiency, procedural knowledge and conceptual understanding for all students. As conceptual understanding is achieved, the interaction may no longer be a key factor. Or, perhaps the interaction is not necessary in conceptual development for students who can independently organize and use their language ability and procedural knowledge. Further research is needed to explore these possibilities.

Another theoretical question raised by this study is whether metacognitive strategies involving mathematics language can help students achieve procedural and conceptual knowledge. There is evidence that teaching students to be aware of their thinking processes during problem solving enhances learning (see Romberg & Carpenter, 1986, for a discussion of metacognition and mathematics). But attention has not been paid to comprehension of spoken and written mathematics language. In reading,

comprehension monitoring strategies have been shown to improve reading comprehension (see Weinstein & Mayer, 1986 for a review). Perhaps these, or similar, strategies can be useful in improving comprehension of mathematics language and, in turn, mathematics concepts.

Directions for Future Research. To further investigate the findings from this study, two directions seem evident. First, the study should be replicated with a larger and more representative sample of junior high school students. The second direction is to conduct an experiment to see if instruction in mathematics language and procedure leads to higher conceptual understanding than instruction in procedure alone. The suggestions presented earlier can be used to select instructional variations for the study.

If the findings from these types of studies support the finding that mathematics language facility influences conceptual achievement, the results will have major implications for teacher training, the design of teaching materials and texts, classroom instruction, testing, and evaluation. Furthermore, the importance of language in mathematics learning and instruction directly relates to the design and use of technology in the classroom. For example, computers and interactive videodiscs have the potential to provide mathematics instruction that incorporates both language and procedure and actively involves

the learner in the interaction of the two. The possibility of this much impact on mathematics learning and instruction warrants continued research on the role of mathematics language facility in mathematics achievement.

References

Bishop, A. J. (1983). Space and geometry. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes. New York: Academic Press.

Caldwell, J. & Goldin, G. (1979). Variables affecting word problem difficulty in elementary school mathematics. Journal for Research in Mathematics, 10, 325-336.

Carpenter, T. P., Corbitt, M. K. Kepner, Jr., H. S., Lindquist, M. M., Reyes, R. E. (1980). National assessment: a perspective of students' mastery of basic mathematics skills. In M. Lindquist (Ed.), Selected issues in mathematics education. Berkeley: McCuthan Publishing.

Carpenter, T., Moser, M., & Romberg, T. (Eds.). (1982). Addition and subtraction: a cognitive approach. New Jersey: Lawrence Erlbaum Associates.

Carpenter, T. P., Hiebert, J., Moser, J. M. (1983). The effect of instruction on children's solutions of addition and subtraction word problems. Educational Studies in Mathematics, 14, 55-72.

Clement, J. (1982). Algebra word problem solutions: thought processes underlying a common misconception. Journal for Research in Mathematics Education, 3, 16-30.

Davis, R. B. & McKnight, C. (1980). The influence of semantic content on algorithmic behavior. Journal of Mathematical Behavior, 3, 39-89.

Davis, R. B. (1984). Learning mathematics: the cognitive science approach to mathematics education. New Jersey: Ablex Publishing Co.

Dudley, A. (1984). The effects of time and data permutation on the solution strategies of first grade children. Focus on Learning Problems in Mathematics, 6, 51-78.

Earp, W. & Tanner, F., (1980). Mathematics and language. Arithmetic Teacher, 28, pp. 32-34.

Fey, J. T. (1970). Patterns of verbal communication in mathematics classes. New York: Teachers College Press.

Gagne, R. M. & Smith, E. C. (1962). A study of the effects of verbalization on problem solving. Journal of Experimental Psychology, 63, 12-18.

Goodlad, J. (1984). A place called school. New York: McGraw Hill.

Hendrickson, D. (1979). An inventory of mathematical thinking done by incoming first-grade children. Journal for Research in Mathematics Education, 10, 7-23.

Hiebert, J. (1982). The position of the unknown set and children's solutions of verbal arithmetic problems. Journal for Research in Mathematics Education, 13, 341-349.

Hiebert, J., Carpenter, T. P., & Moser, J. M. (1982). Cognitive development and children's solutions to verbal arithmetic problems. Journal for Research in Mathematics Education, 13, 83-98.

Hiebert, J., & Wearne, D. (1985). A model of students' decimal computation procedures. Cognition and Instruction, 2, 175-205.

Hoetker, J. & Ahlbrand, W. (1969). The persistence of recitation. American Educational Research Journal, 6, 145-167.

Inhelder, B. & Piaget, J. (1964). The early growth of logic in the child. New York: Harper & Row.

Knight, L. & Hargis, C. (1977). Math language ability: relationship to reading in math. Language Arts, 54, pp. 423-428.

Lesh, R., Landau, M. & Hamilton, E. (1983). Conceptual models and applied mathematical problem-solving research. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes. New York: Academic Press.

Lewis, M. W. & Anderson, J. R. (1985). Discrimination of operator schemata in problem solving: learning from examples. Cognitive Psychology, 17, 26-65.

Linville, W. (1976). Syntax, vocabulary, and the verbal arithmetic problem. School Science and Mathematics, 76, 152-158.

Matz, M. (1980). Towards a computation theory of algebraic competence. Journal of Mathematical Behavior, 3, 93-166.

Mayer, R. E. (1980). Elaboration techniques that increase the meaningfulness of technical text: an experimental test of the learning strategy hypothesis. Journal of Educational Psychology, 72, 770-784.

Nahrgang, C. & Petersen, B. (1986). Using writing to learn mathematics. Mathematics Teacher, 79, pp. 461-465.

National Assessment of Educational Progress (NAEP). (1983). The third national mathematics assessment: results, trends and issues. (Report No. 13-MA-01). Denver, CO: Educational Commission of the States.

Peterson, P. L. & Swing, S. R. (1982). Beyond time on task: students' thought processes during classroom instruction. The Elementary School Journal, 82, 481-491.

Peterson, P. L., Swing, S. R., Braverman, M. T., & Buss, R. (1982). Students' aptitudes and their reports of cognitive processes during direct instruction. Journal of Educational Psychology, 74, 535- 547.

Resnick, L., & Ford, w. (1981). The psychology of mathematics for instruction. New Jersey: Lawrence Erlbaum Associates.

Romberg, T. A. & Carpenter, T. P. (1986). Research on teaching and learning mathematics: two disciplines of scientific inquiry. In M. C. Wittrock (Ed.) Handbook of research on teaching, third edition. New York: Macmillan.

Rosnick, P. & Clement, J. (1980). Learning without understanding: the effect of tutoring strategies on

algebra misconceptions. Journal of Mathematical Behavior, 3.

Siegel, L. S. (1971). The development of certain number concepts. Developmental Psychology, 5, 362-363.

Siegel, L. S. (1982). The development of quantity concepts: perceptual and linguistic factors. In C. Brainerd (Ed.), Children's logical and mathematical cognition. New York: Springer-Verlag.

Skvpek, D. H. (1982). Mathematics education research: the role of language in mathematics teaching and learning. Investigations in Mathematics Education, 15, 111-v1.

Slesnick, T. (1982). Algorithmic skill vs. conceptual understanding. Educational Studies in Mathematics, 13, 143-154.

Tharp, R., Jordan, C., Gallimore, R. (1984). Product and process in applied developmental research: education and the children of a minority. In M. Lamb, A. Brown & B. Rogoff (Eds.), Advances in Developmental Psychology (Vol. III). New Jersey: Lawrence Erlbaum Associates.

Walker, C. H. & Meyer, B. J. (1980). Integrating information from text: an evaluation of current theories. Review of Educational Research, 50, 421-437.

Watkins, A. (1979). The symbols and grammatical structures of mathematical english and the reading comprehension of college students. Journal for Research in Mathematics Education, 10, 216-218.

Webb, N. M. (1980a). An analysis of group interaction and mathematical errors in heterogeneous ability groups. British Journal of Educational Psychology, 50, 1-11.

Webb, N. M. (1980b). Group composition, group interaction, and achievement in cooperative small groups. Journal of Educational Psychology, 74, 475-484.

Webb, N. M. (1980c). A process-outcome analysis of learning in group and individual settings. Educational Psychologist, 15, pp. 69-83.

Webb, N. M. (1982a). Peer interaction and learning in cooperative small groups. Journal of Educational Psychology, 74, 642-655.

Webb, N. M. (1982b). Student interaction and learning in small groups. Review of Educational Research, 52, 421-445.

Webb, N. M. (1983). Predicting learning from student interaction: defining the interaction variables. Educational Psychologist, 18, pp. 33-41.

Webb, N. M. (1984). Microcomputer learning in small groups: cognitive requirements and group processes. Journal of Educational Psychology, 76, 1076-1088.

Weinstein, C. E. & Mayer, R. E. (1986). The teaching of learning strategies. In M. C. Wittrock (Ed.), Handbook of research on teaching, third edition. New York: MacMillan.

Wittrock, M. C. (1974). Learning as a generative process. Educational Psychologist, 11, pp. 87-95.

Wittrock, M. C. (Ed.). (1986). Handbook of research on teaching, third edition. New York: Macmillan.

Wollman, W. (1983). Determining the sources of error in a translation from sentence to equation. Journal for Research in Mathematics Education, 14, 169-181.